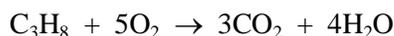


EXAMINATION ONE

I _____ II _____ III _____ IV _____ V _____ Total _____

This exam consists of several questions. Please glance over the entire exam, and then attempt the questions in the order of your choice. **You must show your work to receive any credit for a calculated answer.** Draw a box around your final answer given to the correct number of **significant figures** as defined by our text (Harris 8e), and **be sure to include the correct units.** An information packet was passed out before the exam. Good luck!

- I. (35 points) The fuel used in most “gas” grills is LP gas, where “LP” stands for liquid propane. The formula of propane is C_3H_8 , and the balanced chemical equation for the combustion of propane is given below.



We want to know the “carbon footprint” of a standard LP gas tank.

- A. First you need to model the shape of the tank. What shape are you going to use? Equations to calculate volume for various shapes are given in the info pack.

The best shape to use for the tank is probably a cylinder, but a sphere or even a box would be okay.

- B. Now estimate the dimension(s) you need to calculate the volume and the absolute uncertainty (best guess) in the estimated dimension(s). Give the values in cm. (In case you are better at estimating in inches, remember that 1 in \equiv 2.54 cm.

Representative rough estimates would be 6 ± 2 inches for the radius and $18 \text{ foot} \pm 4$ inches for the height.

radius: $6 \text{ in} \times 2.54 \text{ cm/in} \pm 2 \text{ in} \times 2.54 \text{ cm/in} = 15 \pm 5 \text{ cm}$

height: $18 \text{ in} \times 2.54 \text{ cm/in} \pm 4 \text{ in} \times 2.54 \text{ cm/in} = 50 \pm 10 \text{ cm}$

- C. Calculate the volume of the tank using the dimension(s) you provided in Part B.

$$V = \pi r^2 h = \pi (15 \text{ cm})^2 50 \text{ cm} = 3.5 \times 10^4 \text{ cm}^3$$

- D. Calculate the absolute uncertainty in for the volume you calculated in Part C using the uncertainties you provided in Part B.

From Table 3-1 in Harris 8e (included with the info pack), we see $y = x^a \Rightarrow \%e_y = a\%e_x$

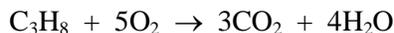
Using that expression to calculate the percent uncertainty in the volume gives

$$\%e_V = \sqrt{(\%e_{r^2})^2 + (\%e_h)^2} = \sqrt{(2\%e_r)^2 + (\%e_h)^2} = \sqrt{\left(2\left(\frac{5}{15}\right)100\%\right)^2 + \left(\frac{10}{50}100\%\right)^2} = 70\% \text{ (1 sf)}$$

So the absolute uncertainty in the volume would be $3.5 \times 10^4 \text{ cm}^3 \left(\frac{70}{100}\right) = 2.5 \times 10^4 \text{ cm}^3$

Hence the volume is $(4 \pm 2) \times 10^4 \text{ cm}^3$.

Continuing from the previous page, we are estimating the “carbon footprint” of a standard LP gas tank based on the combustion of propane according to the chemical equation shown below.



D. Write down your answer to Part C from the previous page. $3.5 \times 10^4 \text{ cm}^3$

E. Now use that volume to estimate the mass of propane in a full tank. The density of liquid propane at room temperature is 0.49 g/mL.*

$$3.5 \times 10^4 \text{ cm}^3 \left(\frac{1 \text{ mL}}{1 \text{ cm}^3} \right) \left(\frac{0.49 \text{ g C}_3\text{H}_8}{1 \text{ mL C}_3\text{H}_8} \right) = \boxed{1.7 \times 10^4 \text{ g C}_3\text{H}_8}$$

F. So finally, what is the carbon footprint of a standard LP gas tank. In other words, how many grams of carbon dioxide would be produced if you burned a full tank of LP gas?

$$1.7 \times 10^4 \text{ g C}_3\text{H}_8 \left(\frac{1 \text{ mol C}_3\text{H}_8}{44 \text{ g C}_3\text{H}_8} \right) \left(\frac{3 \text{ mol CO}_2}{1 \text{ mol C}_3\text{H}_8} \right) \left(\frac{44 \text{ g CO}_2}{1 \text{ mol CO}_2} \right) = \boxed{5 \times 10^4 \text{ g CO}_2}$$

*Why so low? Rhetorical question, so you do not need to answer it, but 0.49 g/mL that is a surprisingly low density for a “liquid” hydrocarbon.

- II. (25 points) A rural town decided to add fluoride to its drinking water to reduce tooth decay in the community. The decision was very controversial, however, as too much fluoride can stain or even etch teeth in a process termed dental fluorosis, and some folks even claimed that fluoridation of water was a communist plot. After a heated city council meeting, it was finally decided to maintain a fluoride concentration of 1.6 ppm (same value recommended in Q10 from HW 00), and that the concentration of fluoride would be monitored on a monthly basis.

One month the fluoride concentration averaged 1.9₆ ppm with a standard deviation of 0.1₈ ppm, based on six separate measurements of the concentration.

What is the 95% confidence interval for the concentration of fluoride that month?

Degrees of freedom = $n - 1 = 6 - 1 = 5$. From Table 4-2 in the info pack, $t_{df=5}^{95\%} = 2.571$.

$$\mu = \bar{x} \pm \frac{ts}{\sqrt{n}} = 1.9_6 \pm \frac{2.571 \times 0.1_8}{\sqrt{5}} = 1.9_6 \pm 0.2_1$$

Do the results differ from the mandated 1.6 ppm at the 95% confidence level? yes

You are the only chemist in this rural community. The fluoride was added to the water as NaF. How you would determine the amount of NaF you would add to the water supply to achieve a concentration of 1.6 ppm, corresponding to 1.6 g of fluoride in 1000 L of drinking water. What other information do you need? Describe the calculation or set it up, but you do not carry out the calculation.

We need to know volume of the water supply and the mass percent fluoride in NaF (which can be calculated from the atomic masses). Determine the mass of NaF required to deliver 1.6 g of fluoride [$1.6 \text{ g} \times (\text{molar mass NaF}/\text{molar mass F})$], and add that amount for every 1000 L of water to be treated.

III. (20 points) As part of a routine check of water quality, a sample of seawater from Pensacola Beach was collected at 9:00 am on March 25, 2010, and analyzed for total hydrocarbons, giving the following results:

number of measurements	5
mean concentration	37.7 ppb
standard deviation	5.2 ppb

A few months later a decision was made to repeat this analysis at 9:00 am on March 25, 2011. A sample of seawater was taken from the exact same spot and analyzed as before for total hydrocarbons:

number of measurements	9
mean concentration	52.4 ppb
standard deviation	4.8 ppb

A. Do the data indicate that the concentration of hydrocarbon has increased compared to last year at the 95% confidence level? Be sure to show your work. You may assume that the population standard deviations are the same for both sets of measurements.

$$s_{pooled} = \sqrt{\frac{s_1^2(n_1 - 1) + s_2^2(n_2 - 1)}{n_1 + n_2 - 2}} = \sqrt{\frac{5.2^2(5 - 1) + 4.8^2(9 - 1)}{5 + 9 - 2}} = 4.9$$

The calculated t value is then

$$t_{calculated} = \frac{|\bar{x}_1 - \bar{x}_2|}{s_{pooled}} \sqrt{\frac{n_1 n_2}{n_1 + n_2}} = \frac{|37.7 - 52.4|}{4.9} \sqrt{\frac{5 \times 9}{5 + 9}} = 5.4$$

Degrees of freedom = $(n_1 - 1) + (n_2 - 1) = n_1 + n_2 - 2 = 5 + 9 - 2 = 12$. From Table 4-2 in the info pack, interpolating between $df = 10$ and $df = 15$, $t_{df=14}^{95\%} = 2.2$.

Therefore $t_{calculated}(5.4) > t_{table}(2.2)$ and we must reject the null hypothesis, and we cannot say with 95% confidence that the concentrations are not different. In other words, the difference between the concentrations in the two samples is significant at the 95% confidence level.

(Note: You can also conclude that the difference is significant at the 95% confidence because the confidence levels do not overlap, but the differences may be significant even if confidence levels do overlap - see next page.)

B. “You may assume that the population standard deviations are the same for both sets of measurements.”

Or can you? How would you have approached the above question if you were concerned that the above assumption might not be valid? Do not do the calculation or describe the procedure in any detail. Just identify the appropriate course of action.

Use the F test.

C. Why is it important to repeat this measurement at the same time and on the same day?

In order to avoid systematic errors due to natural and/or human causes. For example: tides; variations in temperature with the time of day or season that might affect the evaporation or elimination of hydrocarbons; regular variations in the source of the hydrocarbons such as industrial runoff.

D. Speculate on what event prompted the decision was made to repeat the analysis a year later.

The Deepwater Horizon oil spill that occurred in the Gulf of Mexico following the April 20, 2010, explosion of the Macondo Prospect operated by BP.

StatNews # 73: *Overlapping Confidence Intervals and Statistical Significance*

October 2008

In this issue of *StatNews*, we address the question: can we judge whether two statistics are significantly different depending on whether or not their confidence intervals overlap? The answer is: not always. If two statistics have non-overlapping confidence intervals, they are necessarily significantly different *but* if they have overlapping confidence intervals, it is not necessarily true that they are not significantly different.

We can illustrate this with a simple example. Suppose we are interested in comparing means from two independent samples. The mean of the first sample is 9 and the mean of the second sample is 17. Let's assume that the two group means have the same standard errors, equal to 2.5. The 95 percent confidence interval for the first group mean can be calculated as: $9 \pm 1.96 \times 2.5$ where 1.96 is the critical t-value. The confidence interval for the first group mean is thus (4.1, 13.9). Similarly for the second group, the confidence interval for the mean is (12.1, 21.9). Notice that the two intervals overlap. However, the t-statistic for comparing two means is:

$$t = \frac{17 - 9}{\sqrt{2.5^2 + 2.5^2}} = 2.26$$

which reflects that the null hypothesis, that the means of the two groups are the same, should be rejected at the $\alpha = 0.05$ level. To verify the above conclusion, consider the 95 percent confidence interval for the difference between the two group means: $(17 - 9) \pm 1.96 \times \sqrt{2.5^2 + 2.5^2}$ which yields (1.09, 14.91). The interval does not contain zero, hence we reject the null hypothesis that the group means are the same.

Generally, when comparing two parameter estimates, it is always true that if the confidence intervals do not overlap, then the statistics will be statistically significantly different. **However, the converse is not true.** That is, it is erroneous to determine the statistical significance of the difference between two statistics based on overlapping confidence intervals. For an explanation of why this is true for the case of two-sample comparison of means, see the following link: <http://www.cscu.cornell.edu/news/statnews/Stnews73insert.pdf>

As always, if you have any statistical questions, contact the staff consultants at the Cornell Statistical Consulting Unit.

Author: Andrea Knezevic

(This newsletter was distributed by the Cornell Statistical Consulting Unit. Please forward it to any interested colleagues, students, and research staff. Anyone not receiving this newsletter who would like to be added to the mailing list for future newsletters should contact us at cscu@cornell.edu. Information about the Cornell Statistical Consulting Unit and copies of previous newsletters can be obtained at <http://www.cscu.cornell.edu>).

IV. (20 points) The examination you are taking now is not the best way to measure what you know and what you have learned. Oral exams are better. Consider the totally made up data set below.

Student	written exam	oral exam	difference	$(d_i - \bar{d})^2$
1	83	80	3	0.64
2	96	80	16	190.44
3	65	70	-5	51.84
4	78	80	-2	17.64
5	89	90	-1	10.24
			$\bar{d} = 2.2$	$\Sigma = 270.8$

A. Based on this fake data set, do written exams and oral exams measure the same thing?

$$s_d = \sqrt{\frac{\Sigma(d_i - \bar{d})^2}{n-1}} = \sqrt{\frac{270.8}{5-1}} = 8.228$$

$$t_{\text{calculated}} = \frac{|\bar{d}|}{s_d} \sqrt{n} = \frac{2.2}{8.228} \sqrt{5} = 0.598$$

Looking up t_{table} for 95% confidence and $5 - 1 = 4$ df gives $t_{\text{df}=4}^{95\%} = 2.776$.

So $t_{\text{calculated}} (0.598) < t_{\text{table}} (2.776)$, so we cannot state with 95% confidence that the two methods are different.

B. Show that you know how to complete the table by filling in the last two blanks below.

Student	written exam	oral exam	difference	$(d_i - \bar{d})^2$
6	72	50	22	272.25

Note: you will need to calculate a new value for “d bar.” Do not use the data for student #6 to answer Part A.

$$\text{new } \bar{d} = \frac{3+16+(-5)+(-2)+(-1)+22}{6} = 5.5$$

$$(d_i - \bar{d})^2 = (22 - 5.5)^2 = 272.25$$