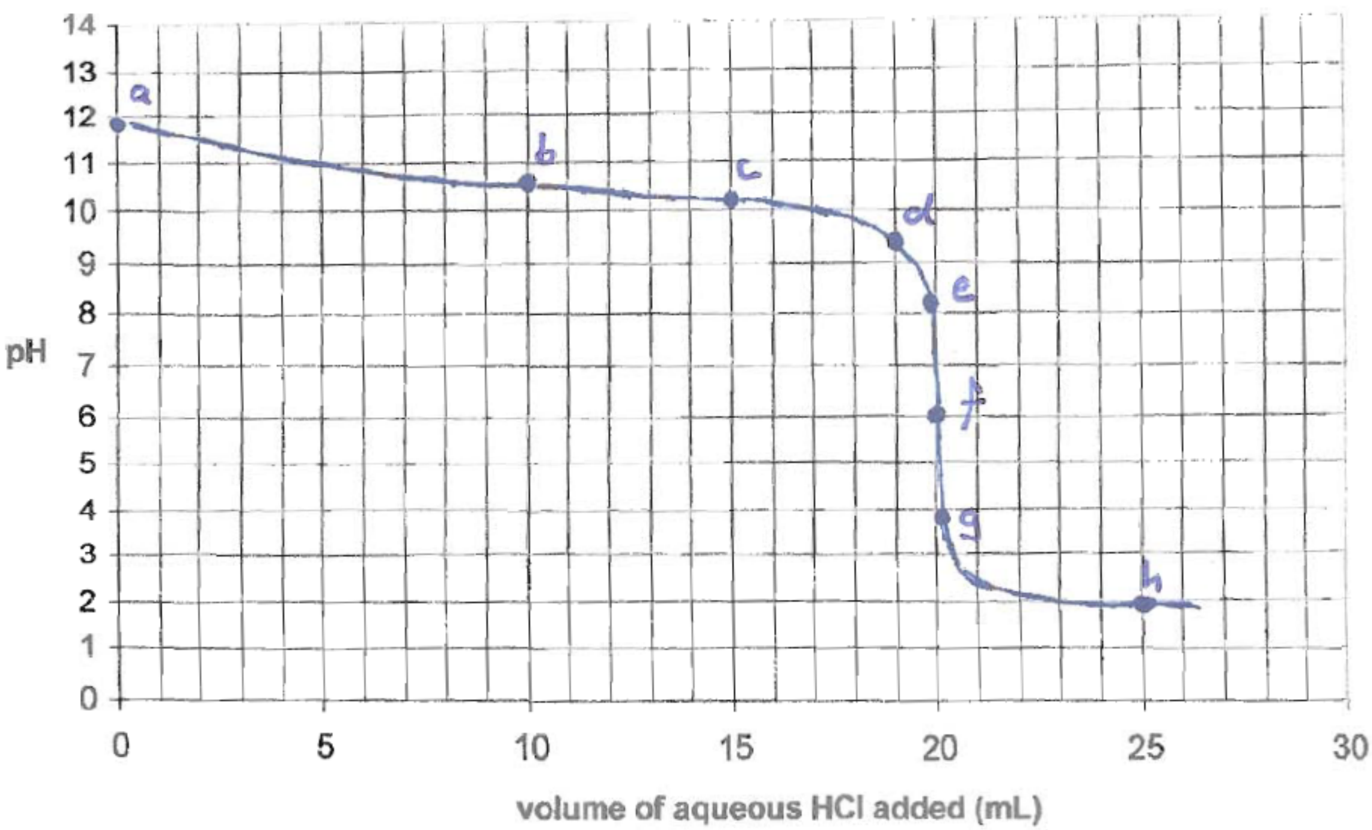
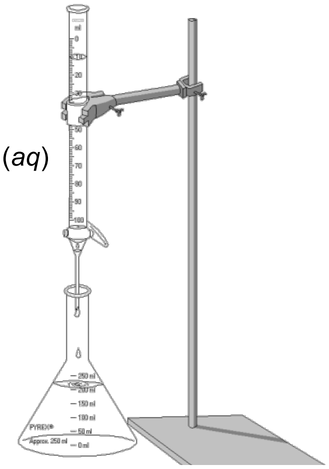
Practice Exercise: Titration of a Weak Base with a Strong Acid – Solutions

A 20.00-mL sample of 0.1000 M $N(CH_2CH_3)_3$ is titrated with 0.1000 M HCl. K_b for $N(CH_2CH_3)_3$ is 5.3×10^{-4}, so the K_a for $HN(CH_2CH_3)_3^+$ is 1.9×10^{-11} and $pK_a = 10.72$. Determine the pH after the following volumes of HCl are added:

a. 0 mL
b. 10.00 mL
c. 15.00 mL
d. 19.00 mL
e. 19.95 mL
f. 20.00 mL
g. 20.05 mL
h. 25.00 mL
i. 30.00 mL

and draw a smooth line on the grid below illustrating the titration curve you would expect for this titration. Circle the "buffer region." The K_a of $HN(CH_2CH_3)_3^+$ is 1.9×10^{-11}.





a. 0 mL of 0.1000 M HCl(aq) added

This is a K_b problem

	$N(CH_2CH_3)_3$ + H_2O <=>	$HN(CH_2CH_3)_3^+$	+ OH^-
initial	0.1 M	--	--
change	– x	+x	+x
equil.	0.1 – x	x	x

Solving

$$5.3 \times 10^{-4} = \frac{x^2}{0.10 - x}$$

gives x = $7.2_8 \times 10^{-3}$ = $[OH^-]$* ⇒ pOH = 2.14 ⇒ pH = 11.86

(*should iterate to get a better answer; 2nd iteration, $7.0_1 \times 10^{-3} = [OH^-]$ ⇒ pH = 11.85)

b. 10.00 mL of 0.1000 M HCl(aq) added

Between the no HCl added and the equivalence point (20 mL), so both $N(CH_2CH_3)_3$ and $HN(CH_2CH_3)_3^+$ present in solution. Therefore we can use the H-H equation.

	$N(CH_2CH_3)_3(aq)$ +	$H^+(aq)$ →	$HN(CH_2CH_3)_3^+(aq)$
before	0.020 L × 0.10 M 0.0020 mol	0.010 L × 0.10 M 0.0010 mol	
change	–0.0010 mol	–0.0010 mol	+0.0010 mol
after	0.0010 mol	--	0.0010 mol

The H-H equation gives

$$pH = pK_a + \log\frac{[B]}{[B\text{-}H^+]} = 10.72 + \log\frac{\text{mol B} / \cancel{\text{total vol}}}{\text{mol B-H}^+ / \cancel{\text{total vol}}} = 10.72 + \log\frac{0.0010}{0.0010} = 10.72 + 0 = 10.72$$

and pH = 10.72 = pK_a as expected for the half-equivalence point.

c. 15.00 mL of 0.1000 M HCl(aq) added

By analogy with part (b)

$$pH = pK_a + \log\frac{[B]}{[B\text{-}H^+]} = 10.72 + \log\frac{\text{mol B} / \cancel{\text{total vol}}}{\text{mol B-H}^+ / \cancel{\text{total vol}}} = 10.72 + \log\frac{0.0005}{0.0015} = 10.72 + (-0.477...) = 10.24$$

giving pH = 10.25. Note that the pH decreased with the addition of acid, as expected, but it did not decrease by much, also as expected because we are in the "buffer region."

d. 19.00 mL of 0.1000 M HCl(aq) added

$$pH = pK_a + \log\frac{[B]}{[B\text{-}H^+]} = 10.72 + \log\frac{\text{mol B} / \cancel{\text{total vol}}}{\text{mol B-H}^+ / \cancel{\text{total vol}}} = 10.72 + \log\frac{0.0001}{0.0019} = 10.72 + (-1.278...) = 9.44$$

giving pH = 9.44.

e. 19.95 mL of 0.1000 M HCl(aq) added

$$pH = pK_a + \log\frac{[B]}{[B\text{-}H^+]} = 10.72 + \log\frac{\text{mol B} / \cancel{\text{total vol}}}{\text{mol B-H}^+ / \cancel{\text{total vol}}} = 10.72 + \log\frac{0.05\times10^{-4}}{19.95\times10^{-4}} = 8.12$$

giving pH = 8.12.

f. 20.00 mL of 0.1000 M HCl(aq) added

This volume takes us to the equivalence point, so all $N(CH_2CH_3)_3$ has been converted to $HN(CH_2CH_3)_3^+$, and we essentially have a solution of $HN(CH_2CH_3)_3^+$. Therefore we must solve a K_a problem where we obtain K_a from K_b

$K_a = K_w/K_b = (1.0 \times 10^{-14}/5.2 \times 10^{-4}) = 1.9 \times 10^{-11}$. The total volume is 40 mL, so $[HN(CH_2CH_3)_3^+] = (½)0.10$ M = 0.050 M. Solving

$$1.9\times10^{-11} = \frac{x^2}{0.05 - x}$$

gives $x = 9.8 \times 10^{-7} = [H^+] \Rightarrow pH = 6.01$

g. 20.05 mL of 0.1000 M HCl(aq) added

This volume puts us past the equivalence point, so we must solve an excess H^+ problem.

0.05 mL = 5×10^{-5} L xs OH^- × 0.10 mol/L = 5×10^{-6} mol H^+

volume = 20.00 mL + 20.05 mL = 40.05 mL = 0.04005 L

$(5 \times 10^{-6}$ mol)/(0.04005 L) = 0.0001_2 M $H^+(aq) \Rightarrow pH = 3.9$

h. 25.00 mL of 0.1000 M HCl(aq) added

By analogy with part (g), 5.00 mL xs OH^- past equivalence point

5.0 mL = 5.0×10^{-3} L xs OH^- × 0.10 mol/L = 5.0×10^{-4} mol H^+

volume = 20.00 mL + 25.00 mL = 45.00 mL = 0.045 L

$(5.0 \times 10^{-4}$ mol)/(0.045 L) = 0.011 M $H^+(aq) \Rightarrow pH = 1.95$

i. 30.00 mL of 0.1000 M HCl(aq) added

10.00 mL xs OH^- past equivalence point

10.0 mL = 1.0×10^{-2} L xs OH^- × 0.10 mol/L = 1.0×10^{-3} mol H^+

volume = 20.00 mL + 30.00 mL = 50.00 mL = 0.050 L

$(1.0 \times 10^{-3}$ mol)/(0.050 L) = 0.020 M $H^+(aq) \Rightarrow pH = 1.70$